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Evaluation of Data Treatment Techniques for Improved Analysis of Fingerprint Images

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ABSTRACT: We evaluated four different methods for data treatment of fingerprint images. The first two methods (Lee's method, and Iterative Automated Noise Filtering method) are used to remove signal-independent additive noise from digital images of fingerprints. The two other methods (low-pass and high-pass wavelet filtering), based on the wavelet transform, are employed not only to reduce the noise, but also to enhance the information of the fingerprints, especially details at the ridges. To evaluate these four filtering techniques we analyzed images of various fingerprints provided by the Police Department of Knoxville, Tennessee, and recorded digitally in the laboratory using inexpensive apparatus. Experimental results show that the methods are effective in diverse cases and have potential in forensic analysis of fingerprints.

KEYWORDS: criminalistics, fingerprints, data treatment techniques

Noise filtering is usually the first step in image analysis. The main objective is to improve the image quality without losing important information contained in the original image. Another important goal is to increase the efficiency of subsequent data-treatment processes, such as image segmentation; or, in the case of fingerprints, image compression, and image recognition.

One of the major problems with signal filtering for fingerprint images is the possible loss of the minute ridge details. The fidelity of the ridge details must be conserved following the filter operation. Most of the information in the fingerprint is contained in the ridges. In fact, the most important information, used to determine if one fingerprint matches another, is not contained in the size of the fingerprint or in the main ridges, but in the minor details of the ridges. These ridges are small, have discontinuities, endings and bifurcations, which make them difficult to distinguish from noise containing similar spacial frequencies. It is therefore important to image filters carefully, especially when the source image has limited resolution due to the image capture equipment.

Although various image restoration and enhancement methods have been proposed for removing degradations in digital images [*1–12*], very few investigators have reported results of comparison between the relatively new wavelet methods and other filters commonly applied to fingerprint images. In this study, we evaluate four different data treatment techniques for digital images of fingerprints;

1) Lee's method with spacial contrast enhancement,
2) Sari-Sarraf's Iterative Automatic Noise Filtering method,
3) a low-pass wavelet transform based filtering method, and
4) a high-pass wavelet-based filtering method.

All these methods are effective, especially the wavelet methods, which are relatively fast and allow the use of wavelet-based compression algorithm, or wavelet image recognition algorithm almost simultaneously (for more information on wavelet compression and recognition of fingerprint images see references [*27*] and [*28*]).

Methodology

Usually most of the additive noise filtering approaches employ the Fast Fourier Transform (FFT), convolution methods, or recursive algorithms [*1–6*]. The algorithms investigated here deviate from these approaches, in making use of either the local statistics [*7–12*] or the wavelet transform [*13–17*]. The first technique, developed by J. S. Lee [*9*], is based on the use of the local statistics of the image (the local mean and local variance). The principle of this technique is relatively simple: the local statistics of the undegraded image are assumed to be equal in a given neighborhood. The value of a pixel in the estimated image is calculated as the difference between the statistics of the corrupted image and that of the noise. This method had been proven to be a very effective tool in noise filtering but one of its major drawbacks is the necessity of using *a priori* knowledge or separate methods to estimate the noise statistics. To correct this drawback, we used a second method, developed by H. Sari-Sarraf and D. Brzakovic [*12*]. Their idea is to use the local statistics in an iterative manner and automatically compute an estimation of the noise statistics from the input image. The third method investigated uses the wavelet transform and is derived from the wavelet-based denoising procedure developed by Donoho and Johnstone [*13–16*] and from denoising and robust wavelet decompositions developed by Bruce and Donoho [*17*]. The fourth method, also based on the wavelet transform, is more effective at enhancing the fingerprint ridges; it uses a decomposition in wavelet subspace to produce a high-pass filter. Finally in the Apparatus section, the apparatus used to record the fingerprint images is described.

Lee's Algorithms

The method developed by Lee [*9*] uses the local statistics of an image to enhance the contrast and filter the noise. Unlike other algorithms, this method does not use any transforms or recursive

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methods; every pixel can be computed independently from its neighbors. These properties lead to a fast digital image processing program.

Before discussing the algorithm itself we define some terms. Consider a digital image X, of size N columns by M rows. The local statistics of the image on a window of size $(2y+1)\times(2z+1)$ could be defined as follows:

$$m_{i,j} = \frac{1}{(2y+1)(2z+1)} \sum_{K=-y}^{y} \left(\sum_{L=-z}^{z} x_{i+K,j+L} \right) \quad (1)$$

$$v_{i,j} = \frac{1}{(2y+1)(2z+1)} \sum_{K=-y}^{y} \left(\sum_{z=-L}^{z} (x_{i+K,j+L} - m_{i+K,j+L} \right) \quad (2)$$

where x_{ij} is the intensity of X at the pixel coordinates (i,j), m_{ij} is the local mean of the image at the coordinates (i,j), and v_j is the local variance at the coordinates (i,j).

Two variations of Lee's algorithm were investigated, special contrast enhancement and additive noise filtering, the latter being part of the Iterative Automated Filtering.

Spacial Contrast Enhancement—The first idea in the Lee's method is to rescale the gray level and redistribute the histogram of the image in order to increase the details. This method is derived from the algorithm of Wallis [8], providing an algorithm that maintains the local mean of the image but permits its local variance to be modified by a factor based on its original local variance. New pixels values are defined as:

$$x'_{i,j} = m_{i,j} + k(x_{i,j} - m_{i,j}) \quad (3)$$

Where k is the factor modifying the local variance. In Lee's algorithm, for $0<k<1$ the filter is equivalent to a low-pass filter, and for $k>1$, the filter is equivalent to a high pass filter.

Additive Noise Filtering—In the second algorithm, we suppose that the image X is corrupted by a white noise W (of mean μ equal to zero, and of variance σ^2), producing an image Z.

$$Z_{i,j} = X_{i,j} + W_{i,j} \quad (4)$$

where z_{ij} is the intensity of the image Z at the pixel coordinates (i,j), and w_{ij} is the intensity of the noise at the pixel coordinates (i,j).

Since we want to estimate X from Z, we use the same technique as before, but this time the factor k depends on the local variance of Z and X as follows:

$$k_{i,j} = \frac{v_{i,j}}{v_{i,j} + \sigma^2} \quad (5)$$

If we replace k in (3) we obtain

$$x'_{i,j} = m_{i,j} + k_{i,j}(x_{i,j} - m_{i,j}) \quad (6)$$

A problem with this method is that we need to know, or estimate the noise variance σ^2. One solution is to use Sari-Sarraf's Iterative Automated Noise Filtering algorithm to estimate the variance.

Iterative Automated Noise Filtering (IAF)

This method developed by H. Sari-Sarraf and D. Brzakovic [11], is a fully automated method that can reduce additive white noise. The principle is similar to that of Lee's method, but it assumes that all the noise cannot be removed in one operation. The algorithm computes an estimation of the variance of the noise and applies Lee's filter until the estimated noise is sufficiently small.

To estimate the noise, the method uses two different computation procedures. At first, we suppose that a certain fraction, K, of the image is of uniform intensity, and we estimate the noise variance from those pixels by the formula:

$$\sigma_1^2 = \frac{1}{MNK} \sum_{i=0}^{MNK} var_1(i) \quad (7)$$

where var_1 is an array containing all the local variances of the image in increasing order, and the sum goes from O to M times N times K.

In the subsequent iteration, we suppose that the noise is always smaller than in the preceding iteration; the noise, in this case, is estimated by:

$$\sigma_j^2 = \text{O} \frac{1}{(i-1)} \sum_{i=0}^{i-1} var_j(i) \quad (8)$$

where I is the position of the first local variance superior to σ^2_{j-1}.

In [11] it was shown that at lim $j \to \infty$ $\sigma_j^2 = 0$; therefore, we stop the algorithm procedure when $\sigma_j^2 = 0$ is reached, and we can consider that the noise is suppressed. This method is relatively effective, but takes a long time for computation especially for large images. This method is the slowest of all the methods investigated.

Low-pass Wavelet Filter

The next two methods use the wavelet transform (WT), which involves the multiresolution signal decomposition developed by Mallat [24]. Here we present a brief explanation of the wavelet transform (for more detail, consult [18–26]).

Wavelet Transform: A Brief Introduction—The wavelet transform (WT) was first introduced by Morlet in [18]. The mathematical background of the WT was developed by the investigators of the "French School" in 1984 and 1989 [19,20]. Meyer showed the connections of the wavelet theory with some earlier results in operator theory [21]. He and Stronberg also found a new family of wavelets, the orthonormal wavelets. Daubechies exploited this idea and created an algorithm to construct compactly supported orthonormal wavelets, based on iteration of discrete filters [22,23], and Mallat introduced the concept of multiresolution analysis [24–26], that we use in this work.

The wavelet transform of a signal is computed by expanding the signal into a new space defined by the dilatations and translations of a unique function ϕ. A continuous wavelet transform of a signal $s(t)$ in one dimension takes the form:

$$W(\alpha, \tau) = \frac{1}{\sqrt{\alpha}} \int \phi\left(\frac{t-\tau}{\alpha}\right) s(t)dt \quad (9)$$

where ϕ is the analyzing wavelet, α represents a time dilatation and τ a time translation.

In this paper, we are only interested in decomposing the signal using an orthonormal wavelet transformation, which can be defined as follows:

$$W(2^i, 2^i n) = \frac{1}{\sqrt{2^i}} \int \phi\left(\frac{t}{2^i} - n\right) s(t)\, dt \quad (10)$$

by substituting in (9) $\alpha=2^i$ and $\tau=2^in$, where i represents the level or order of transformation.

For some well defined wavelets such as the one created by Daubechies [*23*], ϕ acts as a low-pass filter. That is, the function $W(2^i,2^in)$ is the equivalent of the convolution of $s(t)$ with $\phi(t/2^i)$, where ϕ has a different "width" for each level i. If ϕ is viewed as the impulse response of a filter, then W could be interpreted as a low-pass filtering of s (by a filter h, see Fig. 1*A*), followed by a uniform sampling at the rate 2^i. Consequently, by applying a wavelet transform to s we remove some high-frequency details from the signal, and the higher the level i, the more details are removed. Since we do not want to lose all these details, we define another wavelet "orthogonal" to θ, which when applied to s, recovers the details. Transforming the signal s by the wavelet ψ is then equivalent to convoluting it with a high-pass filter (g in Fig. 1*A*) followed by a sampling at the rate 2^i. Based on these transformations, Mallat proposed to repeat the operation, creating a pyramidal algorithm (see Fig. 1*A*), which allows a useful decomposition of the signal. Source code for fast digital wavelet transformation has been published in reference [*27*].

The same algorithm can be easily extended to two dimensions as shown in Fig. 1*B*. When applied in cascade, the algorithm produces a wavelet transformation of the image that can be used in various applications. It could be represented as three progressions $(D1_n)$, $(D2_n)$, $(D3_n)$, and the coarser level s_α, where $2^\alpha=N$. The inverse wavelet transform allows us to reconstruct the image from its decomposition and is computed in the same manner, except that we use (H, G), the inverse filter of h and g. The algorithms for one- and two-dimensional reconstructions are shown in Fig. 2*A* and Fig. 2*B*.

Low-Pass Filtering—In this method we are not applying the complete pyramidal algorithm to the image, but just the decomposition to the level requested by the user (this saves some computation time). After the decomposition, we transform the result in order to filter the image.

Since the white noise is usually a high-frequency phenomenon, our first attempt was to remove the highest details $(D1_1, D2_1, D3_1, \ldots)$ before the inverse transform, hence suppressing the noise. However, experience showed that even if this method works well on some signals, we lose important details in two-dimension images, especially in the fingerprint images used in this study, since the images have just enough resolution to record the ridge details. Donoho and Johnstone [*14,15*], propose a technique of "denoising by wavelet shrinkage." The principle behind this method is to observe that the noise is not the totality of the details of the transformation as we have first assumed, but actually an

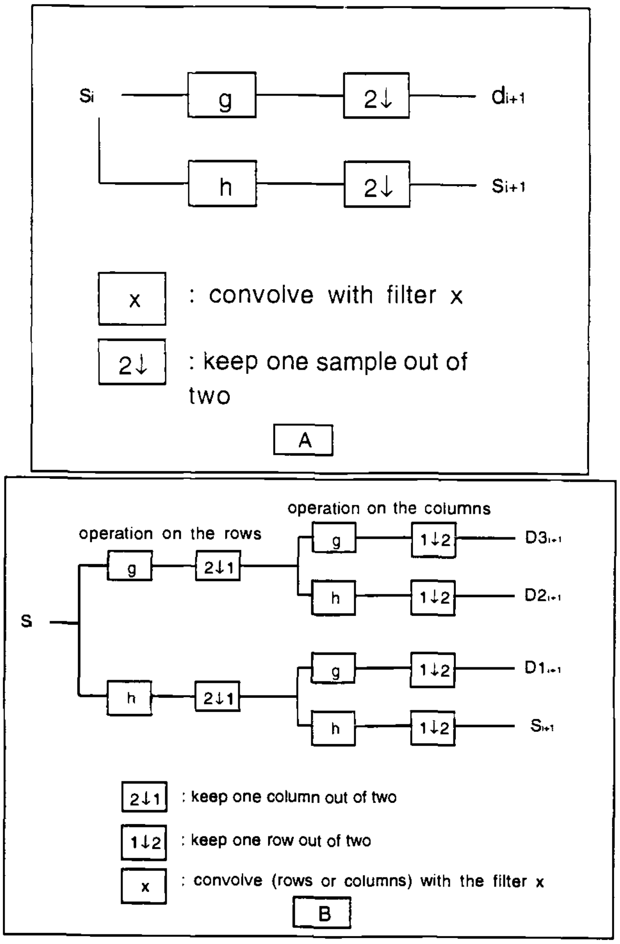

FIG. 1—A) *The decomposition of signal S_i, into the course signal S_{i+1} and detail d_{i+1}, and* B) *the decomposition of an image S_i into S_{i+1}, $D1_{i+1}$, $D2_{i+1}$ and $D3_{i+1}$. The process is repeated in a cascade to compute the wavelet transform.*

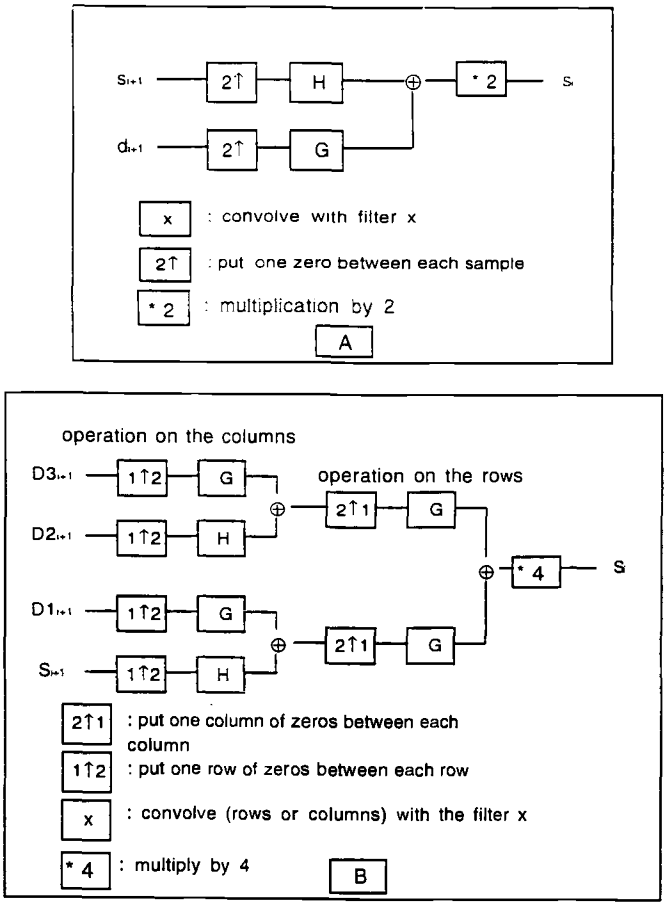

FIG. 2—A) *The reconstruction of S_i from the course signal S_{i+1} and the detail d_{i+1}, and* B) *reconstruction of an image S_i from S_{i+1} $D1_{i+1}$ $D2_{i+1}$ and $D3_{i+1}$. The process is repeated in a cascade to compute the inverse wavelet transform.*

important part of them. The smaller coefficients of the details can be considered as the noise, while the larger ones as the true details. We then apply a transformation that allows reduction of the noise while retaining actual information; we thus shrink the details. There are several possible "Shrink functions;" but, the one specific function we found to work is defined as:

$$D_i' = \text{Shrink}(D_i) = \begin{cases} 0 \quad \text{if} \quad |d_{i,j}| < \lambda_i \\ \text{sign}(d_{i,j})(|d_{i,j}|\lambda_i) \text{ if } |d_{i,j}| > \lambda_i \end{cases} \qquad (11)$$

where λ_i is a value which varies from level to level. After this Shrink transformation is applied, we reconstruct the image using the inverse wavelet transform, and obtain the filtered version of the image.

High-Pass Wavelet Filtering

This method also uses the wavelet decomposition, however, here we apply the algorithm in its totality. To make the equivalent of a high-pass filter using the wavelet decomposition, we first transform the image as follows. We know that the high-frequency details of the image are contained in the higher levels of the wavelet decomposition, so we tried to simply suppress (nullify the value of each point) the lower levels (S_α, $D1_\alpha$, $D2_\alpha$, $D3_\alpha$, . .), but as in the preceding method, the preliminary results were not satisfactory. In a second attempt, we tried to shrink the wavelet transform; but again, the filtering was not efficient. Finally, we applied Lee's algorithm to the lower level of the decomposition (the spacial contrast-enhancement) using a coefficient $k>1$. The value of k changed from level to level. The algorithm, presented in Fig. 7, is described as follows:

1. Select the level of filtering |v|.
2. Apply a wavelet transform to the image.
3. Apply the inverse transform to one level.
4. Use Lee's spacial contrast enhancement filter (with the coefficient k depending on the level).
5. If the level of filtering |v| is reached, finish the reconstruction of the image and stop; else return to step 3, as required.

This method is comparatively slow. However, it takes less time than the Iterative Automatic Noise Filtering method.

Apparatus

The system used to digitally record the fingerprints consisted of a charged-coupled device (CCD, model ST-6, Santa Barbara Instrument Group) camera fitted with a 105-mm macro lens. The camera was held 12–20 inches above the developed fingerprints and a variety of light sources were used to expose the prints. The camera's microcontroller was linked to a desktop computer via a RS-232 serial port. Figure 3 is a schematic of the camera system.

The ST-6 is an inexpensive camera marketed by Santa Barbara for use by amateur astronomers. The entire system, including the computer, costs less than $5000. This camera has a built-in two-stage cooling capability that reduces the dark noise and allows long exposures for increased sensitivity to low-light subjects, such as latent prints developed with fluorescent stains and viewed through spectrally narrow optical filters. The resolution of the camera was 375 by 242 pixels, adequate to record the ridge detail of the prints. While a higher resolution would be useful to allow more effective digital filtering, this would increase the cost of the system. One drawback of this camera is the relatively slow data transfer rate between the microcontroller and the computer; at least 20 *s* are required per image (more if a dark frame is subtracted).

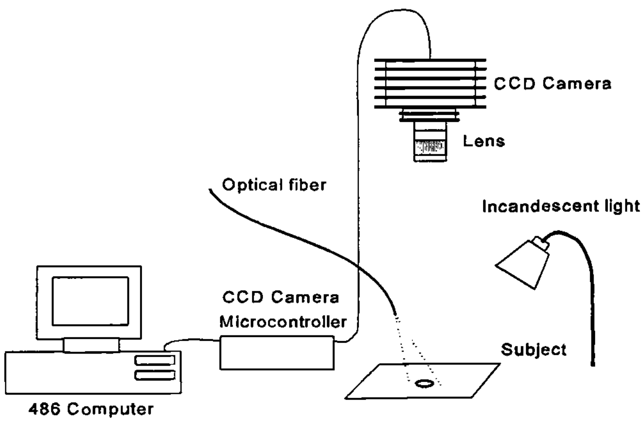

FIG. 3—*Apparatus used to record print images.*

A white light source (incandescent lightbulb) was used to expose most of the prints, and a helium-cadmium laser (325 nm, roughly 4 mW) was used to expose the print developed with fluorescent powder. A 600 μm core glass optical fiber was used to guide the laser light to the print. The fiber was held about 10 inches from the print to allow the cone of light emanating from the fiber to expand to a diameter of 2 cm.

Results and Discussion

We now apply the four methods described previously to the analysis of fingerprint images. All the fingerprint images investigated were acquired using the apparatus described previously. To show the performance of the filtering techniques, we selected six fingerprints with specific problems. Some of these fingerprints (Fig. 4 *A*, Fig. 5 *A*, Fig. 6 *A*) were developed in the laboratory for this study, and others (Fig. 7 *A*, Fig. 8 *A*, Fig. 9 *A*) were provided to us by A. Bohanan of the Knoxville, Tennessee Police Department. The first print (Fig. 4 *A*) was developed with Rhodamine 6G powder. The second print (Fig. 5 *A*) was an impression of an inked finger on paper. The third, fourth, and fifth images (Figs. 6 *A*, 7 *A*, 8 *A*) are very clear fingerprints. The third print (Fig. 6 *A*) was developed with black printer toner powder on a glass slide. The characteristic of the fingerprint image in Fig. 4 *A* is that the image presents high contrasts in intensity. The entire image is a succession of black and white spots in which the ridges appear. The problem with the second image (Fig. 5 *A*) and the sixth image (Fig. 9 *A*) is the opposite; the images show a very low contrast (the background has the same level of intensity as the fingerprint itself), and the signal is significantly corrupted by noise.

The results of filtering are presented in the Figs. 4 to 9. Images labeled 'A' on the upper left corner always represent the original image. Images labeled 'B' show the Lee filtering for almost all the fingerprints (Figs. 4 *B*, 5 *B*, 6 *B*, 7 *B*, 9 *B*), and we can see (particularly on the Figs. 6 *B*, and 7 *B*,) that Lee's algorithm conserves and enhances the ridges already detectable in the original image; Lee's algorithm also increases the resolution in the dark spot of the images (as in Fig. 4*B*), but has no effect on the white spot and the low-contrast region.

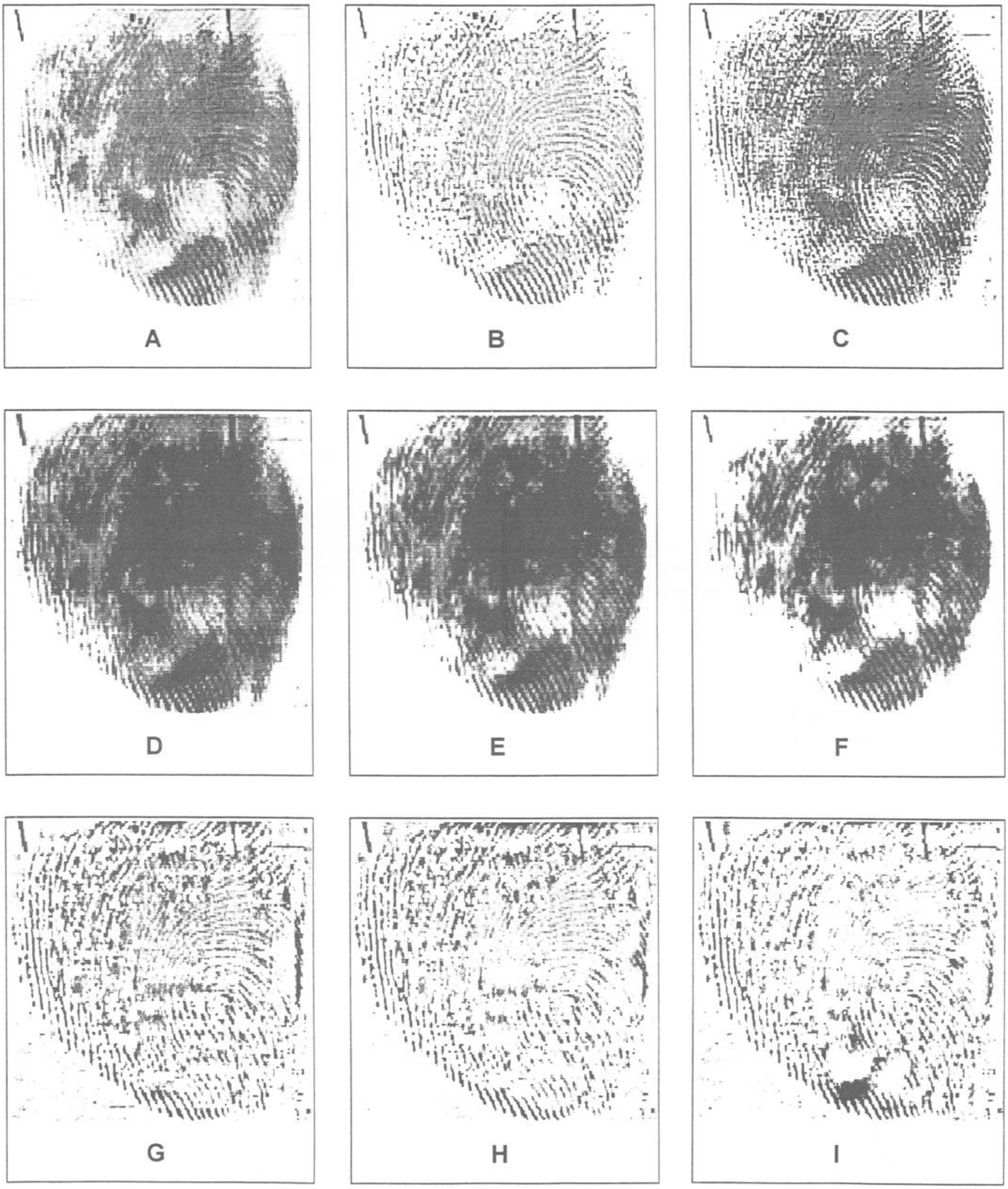

FIG. 4—*Fingerprint on paper with irregular darkness. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Lee filter;* C) *Iterative Automatic Filter;* D-F) *low-pass wavelet filter with Daubechies' wavelets 2, 5, and 10;* G-I) *high-pass wavelet filter with Daubechies' wavelets 2, 5, and 10, respectively.*

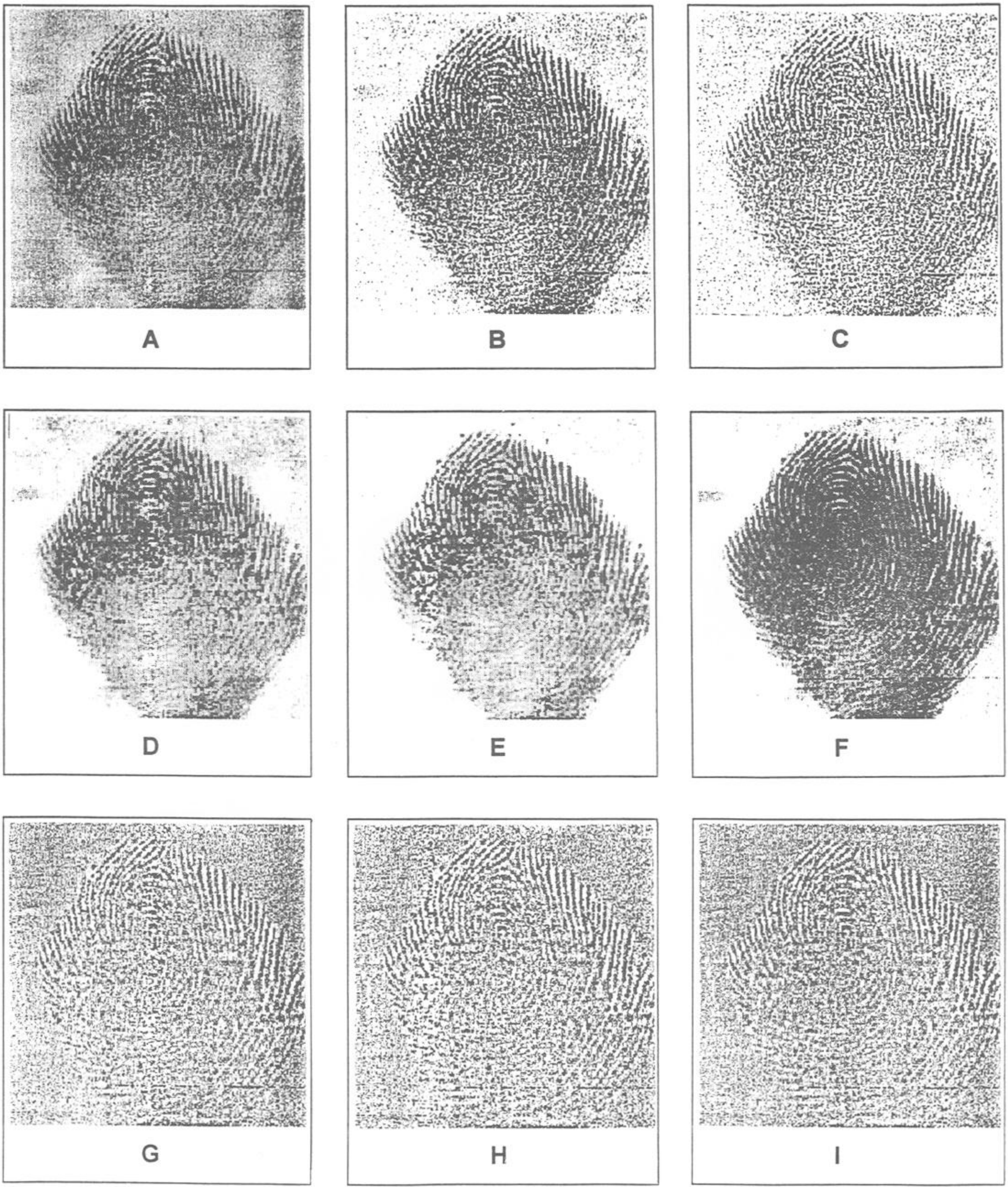

FIG. 5—*Fingerprint on paper with very low intensity. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Lee filter;* C) *Iterative Automatic Filter;* D-F) *low-pass wavelet filter with Daubechies' wavelets, 2, 5, and 10,* G-I) *high-pass wavelet filter with Daubechies' wavelets 2, 5, and 10, respectively.*

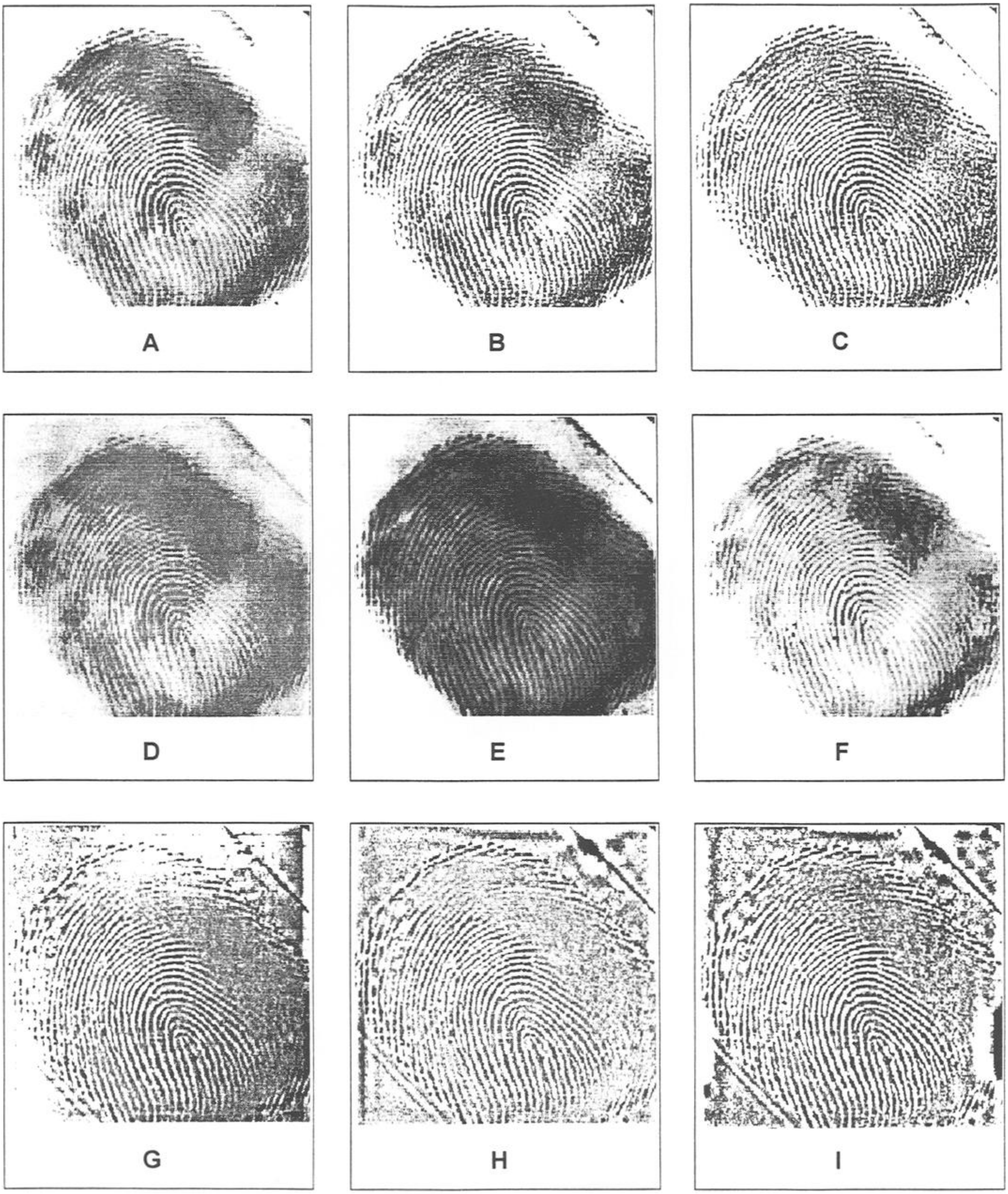

FIG. 6—*Fingerprint on glass developed with toner powder. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Lee filter;* C) *Iterative Automatic Filter;* D-F) *low-pass wavelet filter with Daubechies' wavelets 2, 5, and 10;* G-I) *high-pass wavelet filter with Daubechies' wavelets 2, 5, and 10, respectively.*

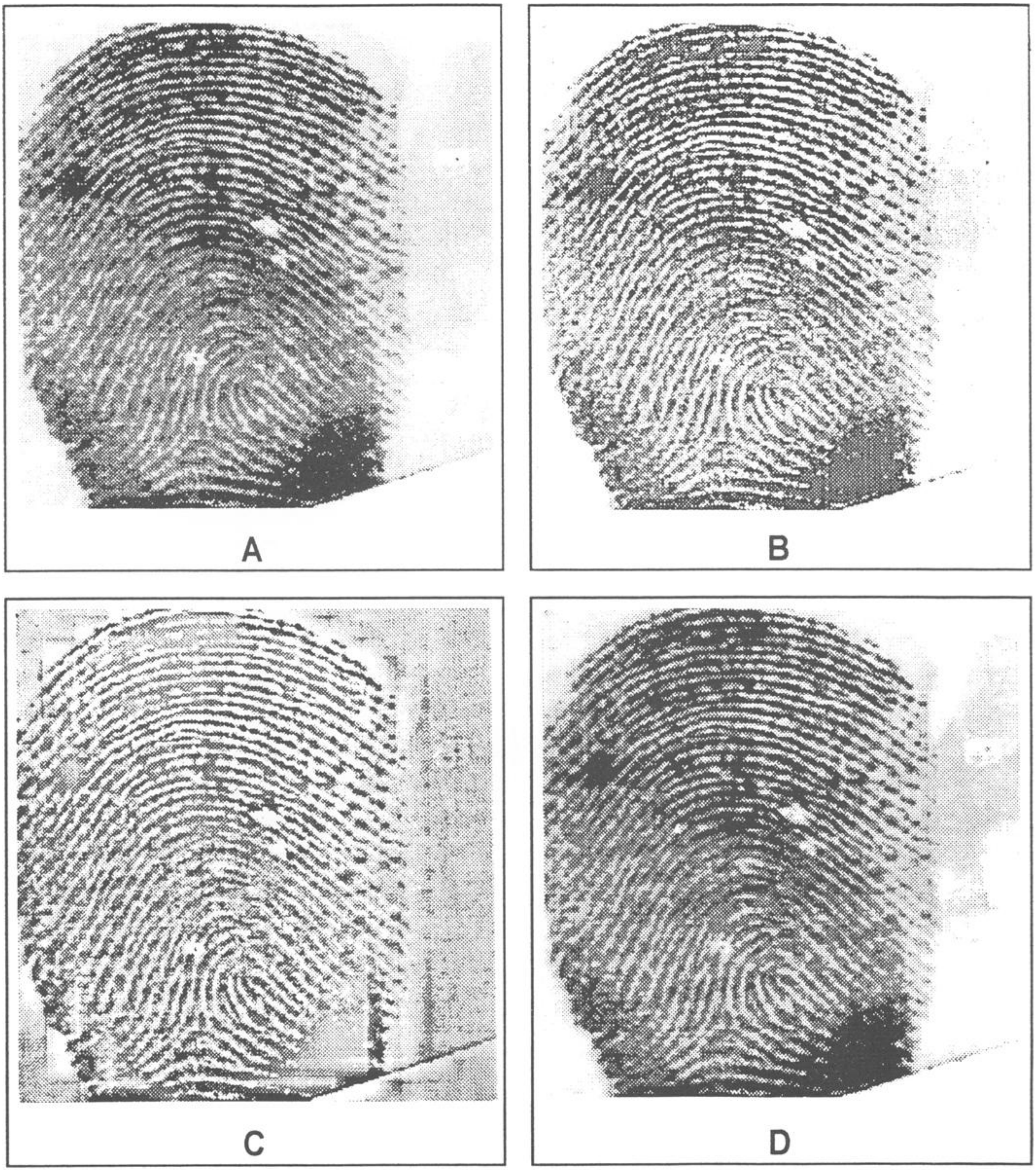

FIG. 7—*Fingerprint on paper. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Lee filter;* C) *Iterative Automatic Filter;* D-F) *low-pass wavelet filter with Daubechies' wavelets 2, 5, and 10;* G-I) *high-pass wavelet filter with Daubechies' wavelets 2, 5, and 10, respectively.*

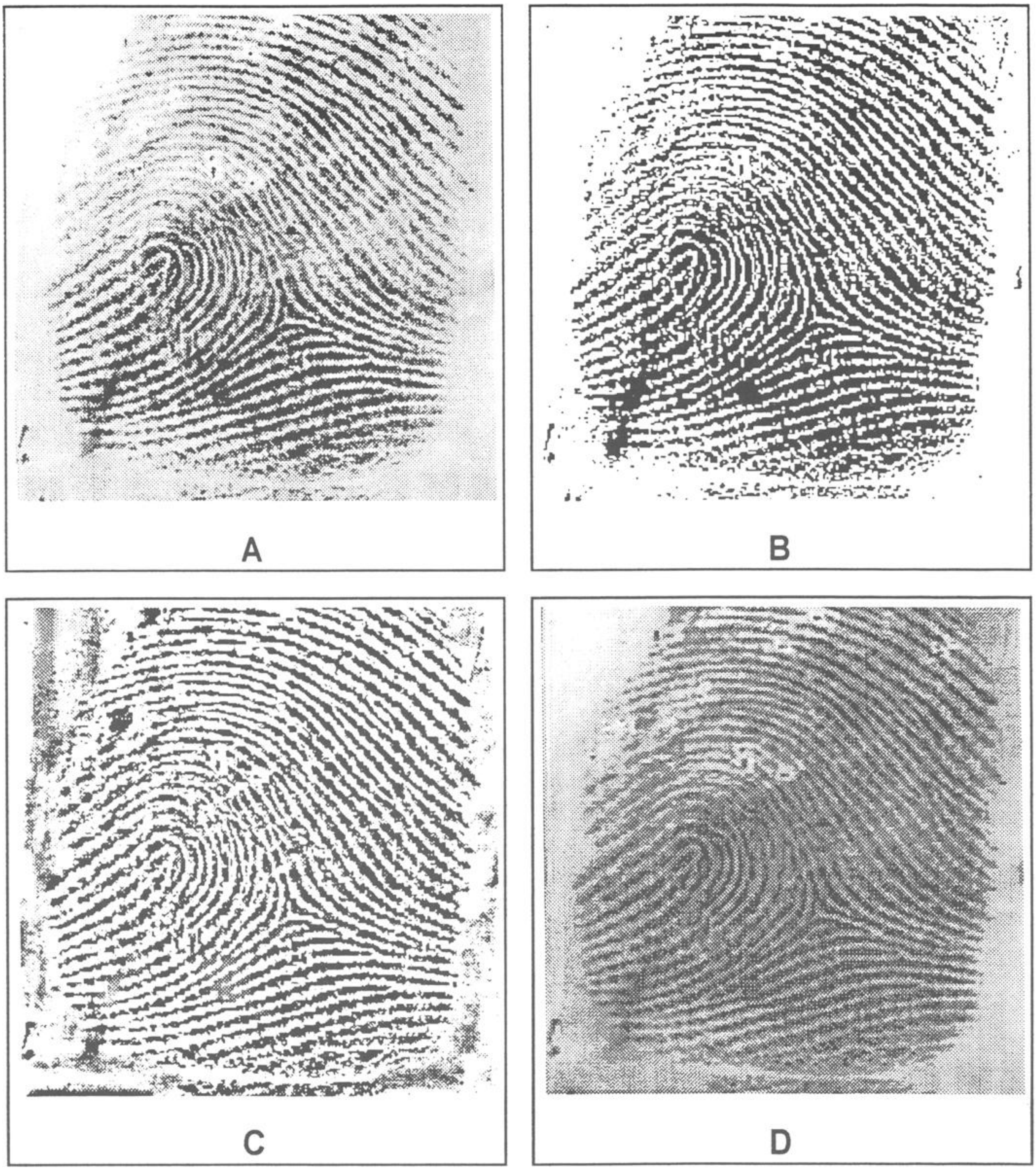

FIG. 8—*Fingerprint on paper. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Iterative Automatic Filter;* C) *low-pass wavelet filter with Daubechies' wavelet 10;* D) *high-pass wavelet filter with Daubechies' wavelet 5.*

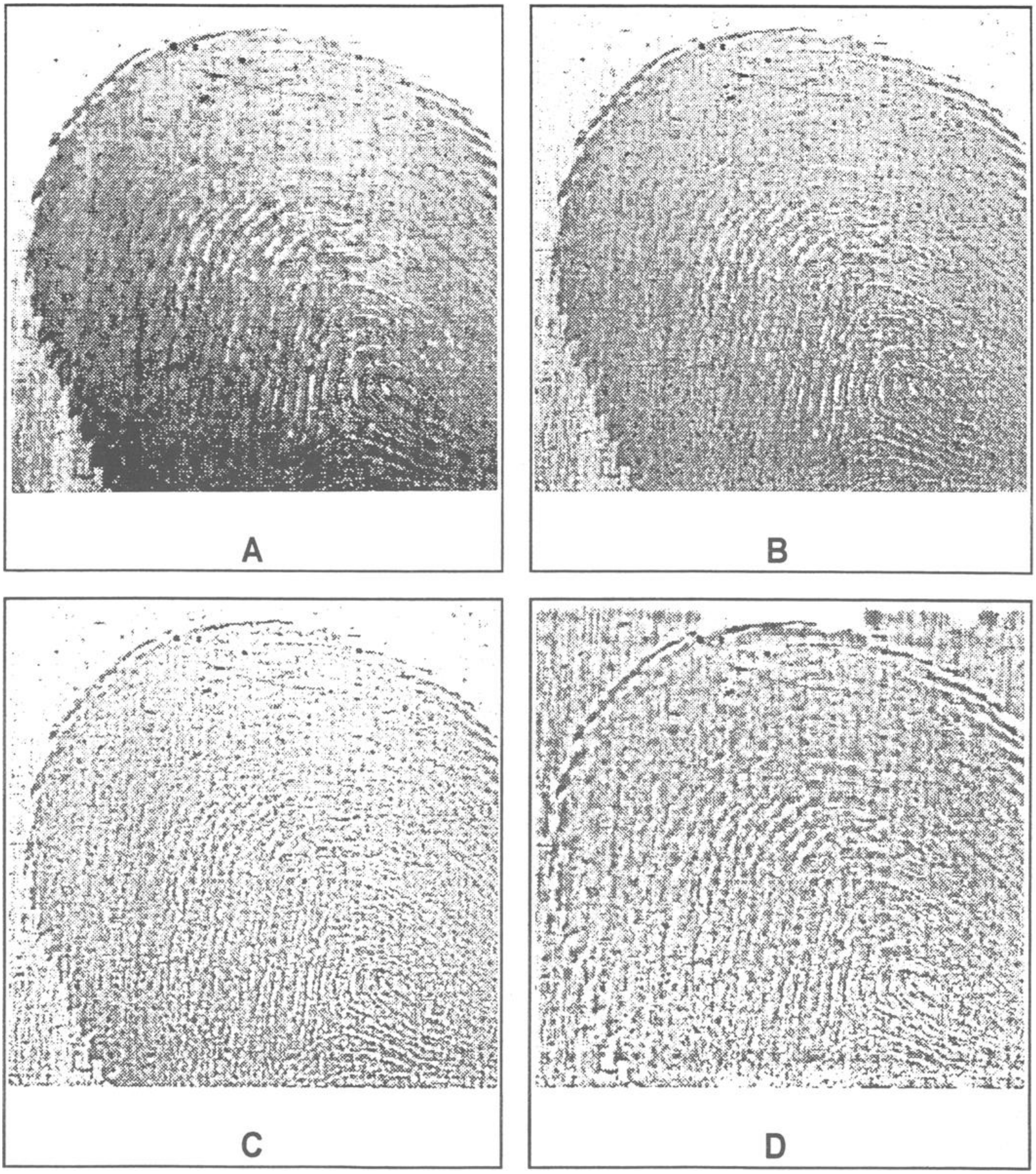

FIG. 9—*Fingerprint on plastic. Image* A *is the original image, the remaining images were treated with the following filters:* B) *Lee filter;* C) *Iterative Automatic Filter;* D) *high-pass wavelet filter with Daubechies' wavelet 10.*

Images 4 *C*, 5 *C*, 6 *C*, 9 *C*, and 8 *B* show the results of the Iterative Automatic Filtering algorithm. This algorithm also enhances the ridges already detectable, especially when the image is clear as in images 6 *C* or 8 *B*. However it aggravates the problem of the dark spots, and does not improve the low-contrast image. The Lee and IAF algorithms could be used to intensify the ridges already detectable in the images. The IAF algorithm is slightly better than Lee's algorithm; but Lee's method, unlike the IAF method, is able to detect some additional ridges in the black areas. Neither of these methods, however, improved the low-intensity (or low contrast) images.

For the two other methods (the wavelet methods), we have chosen to present the results using three different wavelets for the three first fingerprints—to demonstrate that the nature of the wavelet is important in image treatment. Here the wavelets chosen are Daubechies' wavelets 2, 5 and 10, but some other smoothing wavelet may have been chosen. The low-pass filter corrects the noise and is able to increase the resolution of the low contrast image (Figs. 5 *D*, *E*, *F*); it conserves the ridges in the clear image (Figs. 6 *D*, *E*, *F*, or 7 and 8 *C*), but is totally ineffective on the high-contrast image (Figs. 4 *D*, *E*, *F*). This algorithm could be useful for low-contrast images, but is not suitable for the other cases. The last method (high-pass wavelet filtering) is, on the contrary, effective on high-contrast images (Figs. 4 *G*, *H*, *I*). Even the ridges in dark and white spots are enhanced (the smaller wavelet seems to be more effective). The apparent resolution seems to be lower when the image is of low-intensity and corrupted by noise.

The two last methods are complementary: If one method does not improve the image, use of the other will often enhance the image. We can also see that none of the methods are really suitable when there are little features as in the image 9, a fingerprint taken on plastic with a white powder. The performance of the different methods are summarized in Table 1.

Conclusion

In this paper, four methods for reducing noise and for enhancing fingerprint images have been presented. While obtaining marked improvements in these low-resolution images is difficult, the results show that digital filtering can enhance the clarity of important fingerprint features. The results indicate that even if they are not appropriate for every case, the wavelet methods are relatively fast, and could save some additional time, especially when data compression or image recognition is also required. If one uses wavelet methods for both data compression and filtering, much of the total computational time required for wavelet-based data compression has already occurred during the wavelet high-pass filter computation.

TABLE 1—*Summary of the result of the methods on fingerprint images.*

Cases	Lee filter	IAF filter	L-P W filter	H-P W filter
1. Ridges already detectable Figs. 4, 6–8	+	+	o	o
2. Dark spots Figs. 4, 6	+	o	−	+
3. White spots Figs. 4, 7, 8	−	o	o	o
4. Low-contrast Figs. 5, 9	o	o	+	−
5. High-contrast Figs. 4, 7	+	o	−	+
6. Clear image Figs. 6–8	+	+	o	+

NOTE: − = lower quality image, o = same quality image, + = better quality image.

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